MA 533
Spring 2005
Tuesday 6:30-9:00pm.

As for everything else, so for a mathematical theory: beauty can be perceived but not explained.
Arthur Cayley as quoted in J R Newman, *The World of Mathematics* (New York 1956).

Instructor: Dr. Terri Magnus, Regis Hall (top floor)
Contact information: 897-8462, tmagnus@rivier.edu
Office hours: Please let me know whenever you need help. The further you get behind, the harder it is to catch up. You are invited to stop by my office whenever I am there or make an appointment. E-mail questions are encouraged as a means of reaching me between classes and practicing your mathematical writing skills. My scheduled office hours are M 10-11:30 and 3-4, T 4:30-6, W 10-11:30, F 10-11:30, and by appointment. I will also be available just before and after class.

Textbook: *A First Course in Abstract Algebra*, 7th edition, by John B. Fraleigh, Addison Wesley, 2002.
Chapters 1-11,13-15,18-23,26,27,29-32.

Brief course description: This course will focus on the abstract concepts underlying basic algebraic structures. We shall look at several different algebraic systems, many familiar from previous study, and observe some of the commonalities and differences in their algebraic structure. The concept of groups will be defined as a structure with a single binary operation having certain properties and then explored in more depth through such topics as subgroups, cyclic groups, generating sets, permutation groups, direct products, dihedral groups, homomorphisms and isomorphisms, cosets and Lagrange's Theorem, and factor groups. Next algebraic structures with two binary operations will be discussed. These include rings, fields, integral domains, quotient rings, rings of polynomials, factorization of polynomials, and vector spaces. At the end of the course, we shall see how to prove many of the factorization theorems learned in high school algebra by focusing on abstract algebraic structures and their extensions.

Course Objectives:

- To help students recognize algebraic structures as the abstractions of more familiar number systems
- To provide students with a working understanding of groups, rings, and fields
- To help students develop the deeper understanding of algebra needed to teach high school algebra
- To engage students in mathematical thinking, reading, writing, and exploration
- To improve students' proficiency in reading and writing correct proofs
- To develop students' awareness and appreciation of formal axiomatic systems and their applications
- To develop students' ability to write clearly and concisely about mathematical ideas and problem solutions

Teaching Strategies:
- Student reading and writing of mathematics
- Lecture and large group discussions with an expectation of student participation and questioning
- Student presentation of proofs, both orally and in writing
- Active student engagement in group work, discussions, problem solving, and constructive critique
- Homework assignments which encourage both review, exploration, and further growth in understanding

Course Requirements:
- Read assigned sections and attempt problems before coming to class
- Homework collected on a regular basis
- Oral presentation of problems on a semi-regular basis
- Two papers
- Attendance, active participation, and engagement in full-class, small-group, and individual activities
- Two tests and a final exam

Americans with Disabilities Act (ADA): Rivier College wants to provide reasonable accommodations to students with disabilities. To accomplish this goal effectively and to ensure the best use of our resources, timely notice of a disability must be provided to the Office of Special Services for verification and for evaluation of available options. Any student whose disabilities fall within ADA should inform the instructor within the first two weeks of the term of any special needs or equipment necessary to accomplish the requirements for the course. To obtain current information on this procedure, contact the Office of Special Services at telephone extension 8497.

Homework assignments are due at the start of the next class period. Often, these will involve more writing than computations. Your work must be legible and understandable in order to get credit. Please leave ample space (3-inch margin or double spacing) for instructor's comments and questions. Proofs should be written in complete sentences (English and/or symbolic) and include

the statement of the problem or theorem at the beginning. Correct grammar and spelling is expected in proofs and essay questions. The instructor may allow or require a student to rewrite an assignment when she believes that the student will learn from this experience. Attach earlier marked submission to the back of the revision. Initial attempts must be submitted on the due date for full credit.

Tests:. Tests will be scheduled on **February 22** and **April 12**. Final exam will be cumulative and held on **May 3**. Any quiz, which may be unannounced, will count as the equivalent of a homework assignment.

Paper: Students will write two papers during the semester.

The first paper will be an expository or argumentative paper in which each student reflects on why the study of abstract algebra is important for high school teachers. The paper may include a look at the deeper theory behind high school algebra, the role of definitions, reasoning, or proof in high school mathematics, or an introduction of real world applications of abstract algebra which may be accessible to secondary school students. Students may opt to do a lesson plan demonstrating abstract algebra concepts or applications in a secondary school classroom. This paper will be due March 15.

The second paper will include a sequence of proofs of lemmas leading to a major theorem. Due dates and a full assignment description will be distributed once we have covered material (ch. 14) necessary to complete the project.

Assessment and computation of grades:

Homework assignments and quizzes	24%
Class participation, presented problems	12%
Expository paper or lesson plan	8%
Paper with formal proofs	10%
Tests (two)	28%
Final exam	18%

Classroom Policies:

- Active participation requires attendance and arrival to class in time to be prepared for work when the class period begins. Attendance will be taken. If you miss class you are responsible for getting notes from a classmate and notifying the instructor outside of class. Much of the learning will take place in classroom activities that cannot be duplicated easily outside of class. If you miss class, you are responsible for doing all classroom activities you missed, getting the notes from a classmate, and turning in all work on the day it is due. Except in unusual circumstances, students missing more than 20% of the classes (2.8) will fail the course.
- Respect your classmates as well as your instructor. Discussion in class should pertain to the topic of the course and to those asked to speak. All students have a right and responsibility to ask questions and give insight related to the understanding of course content.
- Participation in large and small group discussions is required and assessed for active engagement and contribution. We can all benefit by listening to each other's ideas and approaches. Collaboration is an important skill and is encouraged. The time spent on this course outside of class should average six to twelve hours per week. Some may need to spend more time than this. This includes reading the textbook, reviewing class notes, doing assigned work, working on projects, and preparing for tests.
- All work turned in on tests and quizzes must be entirely your own. On homework, presentations, and papers, whenever your ideas are shaped by a classmate's solution, a discussion with a friend, family member, or faculty, or by a written source, be sure to acknowledge that source in your work. Homework solutions discussed in class should be rewritten in your own words and using your own computations and submitted the following class. Behavior contrary to this will result in a grade of F on the test or assignment. Serious infractions may result in an F for the course.
- If unavoidable circumstances keep you from attending class on the day of the test, you must contact the instructor to explain the absence and, if approved, schedule a make-up. Documentation of the reason for absence and promptness in arranging a make-up is expected.
- E-mail is a great way to keep in touch during the days we don't meet. You will be expected to acquire e-mail access so that the instructor can notify you of any announcements. You will also be encouraged to dialogue with the instructor and other students via e-mail as a means of improving your mathematical writing and understanding.

How to approach problems:

- You should explore each question and write out your thinking in a way that I can follow and you can share with others. Focus on answering the question "why" and communicating your reasons.
- The problems will take time, especially to explore and think. Often you will need to walk away from a problem for a while, and return to the problem for a second or third look before writing it up. Plan on writing rough drafts and ideas first.
- Turn in whatever your thinking is on a question, even if only to say, "I do not understand such and such" or "and then I get stuck here." Even though you may not have a complete answer to a problem, your written comments will be a valuable learning experience. Great mathematics often evolves from initial failure.
- Be as specific as possible. Use pictures. Apply definitions. Identify all variables. Conjecture.

- Accept at the start that learning necessarily entails some struggling with ideas and feelings of frustration. Stay connected with the course and see me whenever you are having difficulty or if extenuating circumstances arise.

The ultimate goal of reasoning is to expand our knowledge, but we cannot seek answers unless we have questions to guide us. We learn, discover, and create by asking questions, so please write down any questions that occur to you as you contemplate the various problems, then bring your written questions to class or e-mail them to me.

Evaluation Criteria for Homework

9.5-10 (A/A+)	Excellent work. You appear to have a very good understanding of the main concepts and procedures in the assignment. Your answers are thorough, well written, and show insight. Proofs are logical, thorough, and precise. Papers are neat and well organized.
8.5-9 (B/AB) (B/B+/A-)	Good work. You completed the assignment and appear to have a good understanding of most of the main ideas or concepts. You may have made a few errors, but not many. Proofs show understanding, are logical, and are for the most part complete, but may need some editing.
7.5-8 (C/C+/B-) (BC)	Satisfactory work. You completed most of the assignment and understand some of the main ideas, but need work in other areas. Some of the problems may have been started, but unfinished. Proofs show some sense of understanding, but you may need to work on style, completeness, logical order, and/or precision. Look over the areas where you had trouble and seek help from the instructor. You may need to be more thorough in your work.
6-7 (D/C-) (C)	Although you made a good attempt at the assignment, your work shows a lack of understanding, sloppiness or carelessness, or inattention to detail. You may need to include more reasons or steps in your answers. Your proofs are sketchy, disorganized, or lack insight. If you had trouble on the assignment, seek help from the instructor. You may have misread the directions for the assignment. Proofs may have been not attempted, incomplete, or showed a lack of understanding and direction.
below 6 (F/D)	Incomplete or poor work. You did not complete most of the assignment. The work that was completed was not well done. You need to include more than just answers in your solutions. If you need help getting started, please ask!
0 (F)	Not enough was done on the assignment to get credit. Perhaps no paper was submitted.

Bibliography:

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Katz, Victor J., *A History of Mathematics*, Harper Collins, 1993.
McCoy, Neal H., and Gerald J. Janusz, *Introduction to Abstract Algebra* (6th), Harcourt Academic Press, 2001.
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