MA 532A Geometric Models for Teachers

Spring 2006
Monday 6:30-9:00pm.

The interplay between generality and individuality, deduction and construction, logic and imagination—this is the profound essence of live mathematics.
--Richard Courant

Instructor: Dr. Terri Magnus, Regis Hall
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Office hours: Please let me know whenever you need help. The further you get behind, the harder it is to catch up. You are invited to stop by my office whenever I am there or make an appointment. I will also respond to e-mail questions. My scheduled office hours are M 3-6, T 2-3, R 9:30-11, F 9:30-11. I will also be available just before and after class.

Textbooks: *College Geometry: A Discovery Approach* (2nd edition) by David C. Kay, Addison Wesley Longman, Inc., 2001.

Materials: Ruler and compass, writing utensil, and plenty of paper. We will be using Geometer's Sketchpad on several nights. A Rivier College computer account is required.

Course description from catalogue: A survey of geometry as a form of axiomatic reasoning; sets of axioms; theorems and the nature of proof; consistency, independence, and completeness of axiomatic systems as demonstrated by geometric models; Euclidean and non-Euclidean models; Hilbert's axioms of neutral geometry; the hyperbolic parallel postulate. Prerequisite: Calculus II, Linear Algebra.

Brief course description: The study of the foundations and applications of Euclidean and non-Euclidean geometries. The course will incorporate cooperative learning, proof-writing, investigations, a project, and use of Geometer's Sketchpad. Topics to be covered include axiomatic systems, incidence geometries, finite geometries, Euclidean geometry, absolute geometry, the parallel postulate, congruence, similarity, area, transformations, and three-dimensional geometry. Students will have an introduction to hyperbolic geometry. In the project, students will investigate and present topics relating geometry to high school teaching.

Course Objectives:
- To explore the rich history of geometry
- To investigate current applications and studies of geometry
- To review high school geometry
- To develop in future teachers the understanding needed to teach geometry effectively
- To engage students in mathematical thinking, reading, writing, and exploration
- To provide students with a variety of tools and methods for analyzing a problem including the use of proofs, models, software, and manipulatives and prepare future teachers to share these tools with their students
- To develop an awareness and appreciation of formal axiomatic systems and their applications
- To develop students' awareness of non-Euclidean geometries and their differences and commonalities
- To develop students' ability to write clearly and concisely about mathematical ideas, proofs, and problem solutions
- To develop students' ability to work together, recognizing that we can all benefit by listening to each other's ideas and approaches and by learning to communicate clearly our own ideas.

Teaching Strategies:
- Student reading and writing of mathematics
- Lecture and large group discussions with an expectation of student participation and questioning
- Laboratory exercises using Geometer's Sketchpad
- Active student engagement in group work, discussions, problem solving, and constructive critique
- Homework assignments which encourage both review, exploration, and further growth in understanding
- Informal and formal proof-writing assignments
- Final project which encourages independent research, organization, and presentation of a related topic

Course Requirements:
- Assigned Reading of Text
- Homework collected on a regular basis
- Oral presentation and/or formal write up of proofs
- Essay on non-Euclidean geometry
- Attendance, active participation, and engagement in full-class, small-group, and individual activities
- Geometer's Sketchpad and manipulative activities
- Two tests

- Portfolio of high school geometry review
- Final project on teaching high school geometry

Homework assignments are due at the start of the next class period. Often, these will involve more writing than computations. Your work must be legible and understandable in order to get credit. Please leave ample space (3-inch margin) for instructor's comments and questions. Rewrites of assignments will be encouraged for additional credit. Correct grammar and spelling is expected in proofs and essay questions.

Tests:. Tests will be scheduled on **February 27** and **May 6**. Final project presentations will be held on **Monday, May 1**. Any quiz, which may be unannounced, will count as the equivalent of a homework assignment.

Papers:

1. An essay on non-Euclidean geometry will be assigned. It will be due on **March 27**.
2. You will be asked to revise one or two proofs from a homework or lab assignment into a professional form based on peer and/or instructor feedback. Details and deadlines will be provided later in the semester.
3. Final project described below.

Review of high school geometry: Graduate students are asked to undertake a review of high school geometry. A high school text will be loaned to you. By **February 20**, you should work out five problems from each of the chapter reviews and self-check your work.

Final project: In lieu of a final exam, graduate students will be required to research a topic related to both geometry and high school teaching. An 8+ page paper will be submitted and a presentation made to the class. Some recommended topics include the following:

- Is the Van Hiele Model of Thinking still important for today's high school geometry teachers? (Be sure to describe the Van Hiele Model) How can you apply this model in your classroom?
- How is an understanding of geometry developed in an integrated math program? Pick one or more of the current IM programs to research. Do students learn more geometry in an IMP or a traditional program? Which do you recommend? What are the benefits and shortcomings?
- Does technology enhance or hinder the study of high school geometry? How often should it be used? In what way(s)? Find arguments on different sides of the issue and weigh the merits of each.
- Explore an application or extension of geometry beyond the typical textbook and our course. Summarize the application and describe how and where you could incorporate it into a high school geometry course. Be sure to identify the mathematical objectives of the lesson.
- Develop your own Geometer's Sketchpad activity and identify how and where you would use it in a high school course. What are the objectives, introductory work, assessment, etc.? In what way does your use of technology enhance the learning of the topic?
- Choose a section from a high school geometry textbook and develop a lesson plan that would encourage students to discover the mathematical theory of that section. Make sure that you're not just presenting the information to them. How do you verify that students fully grasped the main ideas?

Note that for the first four options you must have at least four refereed/published sources. Web journals are acceptable, but other websites need to be approved by the instructor. Separate articles (not letters to the editor or responses) in different issues of the same journal are okay. The eight pages must be your own writing, not quoted statements or activities of others. Brief quotes are allowed, particularly if you wish to contest or support someone's opinion. I want you to demonstrate your geometric understanding and reflect on how it can be applied to the high school classroom. If you choose to do a lesson plan, you must create at least one activity of your own as part of the lesson. You are encouraged to look at published activities for examples of style. Include an introduction that describes why you created this lesson and what activities you created. You may supplement your lesson plan with published activities, activities from a school you've taught at, or material from texts.

Initial proposals of topic are due **February 20**, first versions on **April 3**, and final versions on **April 24**. Projects will presented orally (and visually) during the last class on **May 1**.

Classroom Policies:

- A Rivier College computer account is required as we will be doing Geometer's Sketchpad (GSP) labs in class. If you don't already have one, please visit Information Technology on the first floor in Sylvia Trottier to set one up. You may go to http://www.rivier.edu/it/default.asp?id=40 for more information on Rivier College's Information Technology services and to print a copy of the required form in advance. Some students opt to purchase a copy of GSP for use at home as well, but it is available at most of the college public labs.

- Active participation requires attendance and arrival to class in time to be prepared for work when the class period begins. Attendance will be taken. If you miss class you are responsible for getting notes from a classmate and notifying the instructor outside of class. Much of the learning will take place in classroom activities that cannot be duplicated easily outside of class. If you miss class, you are responsible for doing all classroom activities you missed, getting the notes from a classmate, and turning in all work on the day it is due. If you miss more than two classes, you must meet with the professor to discuss the advisability of your remaining in the course for the remainder of the semester.
- Respect your classmates as well as your instructor. Discussion in class should pertain to the topic of the course and to those asked to speak. All students have a right and responsibility to ask questions and give insight related to the understanding of course content. The instructor may direct a student to discuss questions with her outside of class if she feels it is in the best interest of the class.
- Participation in large and small group discussions is required and assessed for active engagement and contribution. We can all benefit by listening to each other's ideas and approaches. Collaboration is an important skill and is encouraged. The time spent on this course outside of class should average four to eight hours per week. Some may need to spend more time than this. This includes reading the textbook, reviewing class notes, doing assigned work, completing computer assignments, working on projects, and preparing for tests.
- All work turned in on tests and quizzes must be entirely your own. On homework, presentations, and papers, whenever your ideas are shaped by a classmate's solution, a discussion with a friend, family member, or faculty, or by a written source, be sure to acknowledge that source in your work. Behavior contrary to this will result in a grade of F on the test. Serious infractions may result in an F for the course.
- If unavoidable circumstances keep you from attending class on the day of the test, you must contact the instructor to explain the absence and, if approved, schedule a make-up. Documentation of the reason for absence and promptness in arranging a make-up is advised.
- E-mail is a great way to keep in touch during the days we don't meet. You will be expected to acquire e-mail access so that the instructor can notify you of any announcements. You will also be encouraged to dialogue with the instructor and other students via e-mail as a means of improving your mathematical writing and understanding.
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Assessment and computation of grades:

Homework assignments, presented problems, and quizzes	20%
Computer lab reports	10%
NonEuclidean geometry paper	7%
Formal proof assignment(s) and peer critiques	10%
Tests (two)	20%
Review portfolio	3%
Final project	30%

Evaluation Criteria for Homework

9.5-10 (A)	Excellent work. You appear to have a very good understanding of the main concepts and procedures in the assignment. Your answers are thorough, well written, and show insight. Proofs are logical, thorough, and precise. Papers are neat and well organized.
8.5-9 (B/AB)	Good work. You completed the assignment and appear to have a good understanding of most of the main ideas or concepts. You may have made a few errors, but not many. Proofs show understanding, are logical, and are for the most part complete, but may need some editing.
7.5-8 (BC/C)	Satisfactory work. You completed most of the assignment and understand some of the main ideas, but need work in other areas. Some of the problems may have been started, but unfinished. Proofs show some sense of understanding, but you may need to work on style, completeness, logical order, and/or precision. Look over the areas where you had trouble and seek help from the instructor. You may need to be more thorough in your work.
6-7 (C/F)	Although you made a good attempt at the assignment, your work shows a lack of understanding, sloppiness or carelessness, or inattention to detail. You may need to include more reasons or steps in your answers. Your proofs are sketchy, disorganized, or lack insight. If you had trouble on the assignment, seek help from the instructor. You may have misread the directions for the assignment. Proofs may have been not attempted, incomplete, or showed a lack of understanding and direction.
1-5(F/D)	Incomplete or poor work. You did not complete most of the assignment. The work that was completed was not well done. You need to include more than just answers in your solutions. If you need help getting started, please ask!
0 (F)	Not enough was done on the assignment to get credit. Perhaps no paper was submitted.

How to approach problems:

- You should explore each question and write out your thinking in a way that I can follow and you can share with others. Focus on answering the question "why" in explanations and proofs.

- The problems will take time, especially to explore and think. Often you will need to walk away from a problem for a while, and return to the problem for a second or third look before writing it up. Plan on writing rough drafts and ideas first.
- Request help from the instructor by sharing what your attempts are on the problem, "I do not understand such and such" or "and then I get stuck here." Even though you may not have a complete answer to a problem, your written comments will be a valuable learning experience.
- Be as specific as possible. Use pictures. Conjecture.
- Accept at the start that learning necessarily entails some struggling with ideas and feelings of frustration. Stay connected, and see me whenever you are having difficulty or if extenuating circumstances arise.
- The ultimate goal of reasoning is to expand our knowledge, but we cannot seek answers unless we have questions to guide us. We learn, discover, and create by asking questions, so please write down any questions that occur to you as you contemplate the various problems, then bring your written questions to class.

Course Outline and Tentative Schedule:

January 23	1.1,1.3, 2.1, 2.2, 2.3	Discovery, Proof, Incidence Geometries
January 30	2.3, 2.4, 2.5	Incidence, Distance, Angle Measurement
February 6	2.6, 3.1	Plane Separation, Crossbar Theorem, SAS
February 13	3.2, 3.3	Taxicab Geometry, Triangle Congruence
February 20	3.4, 3.5, 3.6, project topic due	Inequality Theorems, Congruence
February 27	3.7, 3.8,test 1	Quadrilaterals, Circles
March 6	Vacation!!	
March 13	6.1, 6.2	History of Non-Euclidean Geometry
March 20	4.1, 4.2	Euclidean Parallel Postulate
March 27	4.2, 4.3, 4.5, non-Euclidean geometry essay due	Parallel Projection , Similar Triangles, Circles
April 3	4.4, 5.2, 5.3, first submission of project due	Tilings by regular polygons, isometries
April 10	5.4, 5.5	Linear Transformations, Coordinate Characterizations
April 17	6.3,6.4,	Hyperbolic Geometry
April 24	7.1, 7.2, final version of project due	Orthogonality, Parallelism in Space, Three dimensions
May 1	project presentations	
May 6	Final test	

Bibliography for Geometric Models:

Classic Texts:

Coxeter, H. S. M. (1969). Introduction to geometry (2nd edition). New York: J. Wiley & Sons.
Greenberg, M. (1972). Euclidean and Non-Euclidean Geometries (2nd edition). New York: W. H. Freeman & Sons.
Hilbert, D. (1971). Foundations of Geometry (2nd edition). La Salle, IL: Open Court.
Moise, E. (1974). Elementary Geometry from an Advanced Standpoint (2nd edition). Reading, MA: Addison-Wesley.

Other Texts:

Adler, C., Modern Geometry.
Baragar, Arthur (2001) A Survey of Classical and Modern Geometries with Computer Activities, Upper Saddle River, NJ: Prentice Hall.
Blumenthal, L. (1980). A Modern View of Geometry. New York: Dover.
Eves, H. (1994). Geometry. Boston: Jones and Bartlett.
Henderson, D. (1996). Experiencing Geometry on Plane and Sphere. Upper Saddle River, NJ: Prentice Hall.
Hilbert, C. and Cohn-Vossen, S. (1990). Geometry and the Imagination. New York: Chelsea.
Hvidsten, Michael (2006) Geometry with Geometry Explorer, New York: McGraw-Hill.
Holden, A. Shapes, Space, and Symmetry. New York: Dover.
Meserve, B. (1983). Fundamental Concepts of Geometry. New York: Dover.
Ryan, P. (1986). Euclidean and non-Euclidean Geometry. New York: Cambridge University Press.

Sibley, T. (1998). The Geometric Viewpoint. Reading, MA: Addison-Wesley.
Smart, J. (1988). Modern Geometries. Brooks/Cole.
Stahl, Saul (2003) Geometry from Euclid to Knots, Upper Saddle River, NJ: Pearson.
Trudeau, R. (1987). The non-Euclidean Revolution. Boston: Birkhauser.
Venema, Gerard (2006) Foundations of Geometry, Upper Saddle River, NJ: Pearson.
Wallace, E., and West, S. (1998). Roads to Geometry (2nd edition), Upper Saddle River, NJ: Prentice Hall.
Weeks, J. (1985). The Shape of Space. New York: Marcel Dekker, Inc.

Geometry and Cognition, Education Reading:

Clemens, C. H. & Clemens, M. A. (1991). Geometry for the Classroom. New York: Springer-Verlag.
Lindquist, M. (ed.) (1987) Learning and teaching geometry K-12: 1987 yearbook. Reston, Virginia: National Council of Teachers of Mathematics.
National Council of Teachers of Mathematics (1991). Professional Standards for Teaching Mathematics. Reston, VA: NCTM, Inc.
O'Daffer, P. (1981). Geometry: What shape for a comprehensive, balanced curriculum? In M. Lindquist (ed.) Selected Issues in Mathematics Education. Chicago: University of Chicago Press.
Serra, Michael (1989). Discovering Geometry. Berkeley, CA: Key Curriculum Press.
Tietze, M. (1992). A core curriculum in geometry. Mathematics Teacher, 85(4), 300-303.
Van Hiele, P. M. (1986). Structure and Insight. New York: Academic Press.
Zimmerman, W. & Cunningham, S. (eds.) (1991). Visualization in Teaching and Learning Mathematics. Washington, D.C.: Mathematical Association of America.

Alternative Approaches, Related Readings:

Abbott, Edwin A., (1992). Flatland: A Romance of Many Dimensions. New York: Dover Publications, Inc.
Banchoff, R. F. (1990). Beyond the Third Dimension: Geometry, Computer Graphics, and Higher Dimensions. New York: Scientific American Library.
Coxeter, H. S. M. and Greitzer, S. L. (1967). Geometry Revisited. Washington, D. C.: Mathematical Association of America.
Sved, M. (1991) Journey into Geometries. Washington, D. C. :Mathematical Association of America.
Schattschneider, D. (1992). Visions of Symmetry: Notebooks, Periodic Drawings, and Related Work of M.C. Escher. New York: W. H. Freeman and Co.
Toth, F. (1964). Regular Figures. Elkins Park, PA: Franklin Book Company.
Wehl, H. (1952). Symmetry. Princeton, NJ: Princeton University Press.

Proof Writing:

Solow, D. (1990). How to read and do Proofs: An Introduction to Mathematical Thought Processes, 2nd edition. New York: J. Wiley & Sons.
Velleman, D. How to Prove It. New York: Cambridge University Press.

Popular Mathematics Literature, Lower Level Texts

Abbott, E. A. (1984). Flatland: A Romance of Many Dimensions. New York: Penguin Books.
Burger, E. and Starbird, M. (2000). The Heart of Mathematics: An Invitation to Effective Thinking. Emeryville, CA: Key College Publishing.
Courant, R. and Robbins, H. (1941). What is Mathematics? New York: Oxford University Press.
Dewdney, A. K. (1984). The Planiverse: Computer Contact With a Two-Dimensional World. New York: Poseidon Press.
Dunham, William (1994). The Mathematical Universe: an alphabetical journey through the great proofs, problems, and personalities. New York: Wiley & Sons.
Eves, H. (1969). In Mathematical Circles. Boston: Bowker.
Gardner, M. (1967). Knotted Doughnuts and Other Mathematical Entertainments. New York: W. H. Freeman and Co.
Gay, D. (1998). Geometry by Discovery. New York: John Wiley & Sons.
Manning, H. (ed.) (1910). The Fourth Dimension Simply Explained. Munn & Co. (reprinted by Dover, 1977).
Peterson, I. (1988). The Mathematical Tourist: Snapshots of Modern Mathematics. New York: W. H. Freeman and Co.
Peterson, I. (1990). Islands of Truth: A Mathematical Mystery Cruise. New York: W.H. Freeman and Co.
Steen, L. (ed.) (1990). On the Shoulders of Giants: New Approaches to Numeracy. Washington, D.C.: National Academy Press.