A GREAT discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest, but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.

--George Pólya from How to Solve It (Princeton 1945).

Instructor: Dr. Terri Magnus, Regis Hall (first floor)
Contact information: 897-8462, tmagnus@rivier.edu
Office hours: Please let me know whenever you need help. The further you get behind, the harder it is to catch up. You are invited to stop by my office whenever I am there or make an appointment. I will also respond to e-mail questions. My scheduled office hours are T10am-12pm, 5-6 pm, W 9:30-11:30am, and R4-6pm in Regis Hall. I will also be available just before and after class.

Textbook:
✓ For the modeling portion of the course, the text A Course in Mathematical Modeling by Douglas Mooney and Randall Swift, The Mathematical Association of America, 1999, is available from the Rivier College bookstore. You may find it useful to have a Calculus text and a spreadsheet program such as Excel available as well.
✓ For the problem-solving portion of the course, no text is required. It is recommended that students have access to a Discrete Mathematics text such as Rosen’s Discrete Mathematics and its Applications (5th edition) Boston: WCB/McGraw-Hill, 2003. Students not owning a Discrete Mathematics text or wishing to have a text more closely aligned with this course may obtain The Art and Craft of Problem Solving by Paul Zeitz, NY:John Wiley and Sons, Inc., 1999. These texts (4th ed. of Rosen), along with other problem-solving texts, will be available through the library reserve desk.

Brief course description: The problem-solving thread of the course will be devoted to learning a variety of problem-solving heuristics and mathematical problem-solving techniques as students work on problems both individually and in small groups. Students will practice reading problems carefully, determining an appropriate technique for attacking each problem, and writing up their solutions. A number of mathematical areas will be considered including graphical models, integer patterns, pigeon-hole principle, finite differences, combinatorics, geometry, invariant problems, induction/recursion, and the extremal principle.

The second focus of the course will be on mathematical models. After a brief introduction, students will study discrete dynamical systems and determine fixed points analytically and graphically. Fitting curves to data will also be discussed. If time permits, the application of differential equations to mathematical modeling will be explored.

In addition to learning the techniques and concepts of problem solving and modeling, students enrolled in MA508 will be asked to complete two projects in which they integrate these ideas into secondary mathematics education.

Prerequisites: Algebra, precalculus, and calculus. Also a course in discrete mathematics is strongly recommended prior to or concurrent with this course.

Course Objectives:
A student completing this course should be able to...
• read a problem and determine a reasonable approach to solving the problem.
• understand and employ a variety of problem-solving heuristics.
• apply techniques and knowledge from number theory, graph theory, probability, geometry, and other mathematical fields to problem solving.
• write clear and complete solutions to problems.
• demonstrate problem-solving skills to others.
• plan lessons that encourage the development of problem solving and mathematical modeling skills in students.
• develop discrete and continuous mathematical models and use them to understand a phenomenon outside of mathematics.

Teaching Strategies:
• Students actively solving problems individually and in small groups.
• Problem exploration and solving both in class and as homework.
• Types of problems intermixed so that students develop the ability to choose appropriate means to solving them.
• Student write-ups of problem solutions.
• Reflections on problem solving process.
• Student presentation and constructive critique of problem solutions.
• Lectures only as needed to clarify problem solving techniques and heuristics, and to introduce modeling.
• Use of spreadsheets and graphs to observe mathematical behavior.
• Students completing and presenting projects in which they connect the ideas of this course to secondary mathematics education.

Course Requirements:

• Attendance, active participation, and engagement in full-class, small-group, and individual activities.
• Full write-up of a suggested number of mixed-type problem solutions.
• Oral presentation of some solutions to the class.
• Modeling project.
• Problem solving in the mathematics classroom project.
• Homework problems applying and extending the material and techniques covered in class.
• Oral presentation of one of the two projects

Final grades: Students will accumulate points through the various assignments.

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Points</th>
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<tbody>
<tr>
<td>Mixed problems</td>
<td>30</td>
</tr>
<tr>
<td>Focused homework problems</td>
<td>16</td>
</tr>
<tr>
<td>Oral presentations/participation</td>
<td>10</td>
</tr>
<tr>
<td>Written projects (17 points each)</td>
<td>34</td>
</tr>
<tr>
<td>Oral presentation of project</td>
<td>10</td>
</tr>
</tbody>
</table>

A (93-100 points), A- (92-90 points), B+ (89-87 points), B- (80-82 points), C+ (77-79 points), C (73-76 points), C- (70-72 points), D+ (67-69 points), D (63-66 points).

Mixed Problems: Frequently, you will be presented with a list of problems. The problems will be taken from a variety of sources and require a variety of techniques to be solved. These problems will often not be based only on the techniques just discussed in class. You are not expected to solve all of the problems, but should make your best effort to complete and write up some of the solutions each week. Submitted solutions must be legible and include a statement of the problem, the number of the list that contained the problem, and the number of the problem itself. Normally a word or numerical value will not suffice for a solution; you must use full sentences as you demonstrate (prove) why your solution is correct. Problems have been assigned a point value (1 for free throw, 2 for field goal, 3 for beyond the arc) based on the anticipated level of difficulty. You may go back and solve problems from earlier lists, provided the solution has not already been presented to the class. You are also allowed to give alternative solutions to previously presented problems provided that your solution differs substantially from the solution presented earlier. The instructor retains the right to determine when partial solutions deserve credit and when outstanding solutions deserve additional credit. The instructor will give feedback on incomplete solutions and allow resubmissions until someone in the class has solved the problem. Aim to earn approximately 2-3 points per week over the semester. Of course, you are welcome to do more soon!

Homework Problems: After techniques in mathematical problem-solving and modeling have been discussed in class, the instructor may give an assignment related to the material. These problems will be due the following class day. You are encouraged to contact the instructor prior to the due date when you have questions on the assignment. Once again solutions must be legible, complete, and written in full sentences. Good proof writing style is expected. The statement (or your restatement) of the problem should precede the solution. Some homework assignments will take the form of mini-projects and will be weighted as the equivalent of two homework assignments.

Oral presentations of problems: Students will be expected to present some solutions and partial solutions to the class. Classmates will then discuss the presentation considering some of the following questions: Is the solution persuasive? Has the solution been communicated effectively? Is the argument sufficiently thorough? Have all factors been taken into account? Can a more general problem be solved? Are there alternative approaches? Can the proof be simplified? Students may earn additional points by significantly contributing to the discussion. Normally, oral problem presentations will be worth 1-3 points each depending on the difficulty, progress, and thoroughness. Students may also earn points for presenting partial solutions and significant attempts at solving problems. Students should contact the instructor (via e-mail) prior to class indicating their desire to present, their problem of choice, and the extent to which they have solved the problem.

Modeling project: Each student will complete and present a project in which they either (1) choose a situation outside of mathematics (environmental studies, economics, traffic flow, social demographics, etc.) and develop and analyze it using
mathematical modeling techniques or (2) develop a activity to encourage mathematical modeling skills in secondary students. Details of the project requirements are attached, and students will be expected to present their project in writing.

**Problem-solving in the classroom project.** MA525 students will develop a lesson in which they use a non-routine problem in teaching secondary mathematics. Details are attached.

**Final presentation:** In lieu of a final examination, graduate students will give an oral presentation (15-20 minutes) on one of their two projects to the class.

**Extra points:** It is possible for students to exceed the specified number of points in the categories of mixed problems and oral presentations. These points may be used to offset lower totals in other categories provided that the student earns at least half of the points within that category. In particular, every student is expected to give some oral presentations, to complete the two projects, and to do homework assignments, and must earn at least 50% in each of these categories before points beyond 30 from the mixed problems category or beyond 10 from problem presentation category would be allowed to improve the overall average.

**Running Total?** Feel free to e-mail or visit the instructor to verify your current total and distribution of points at any time during the semester. The instructor will keep a chart indicating the problems you have earned points on. You are encouraged to do the same.

**Americans with Disabilities Act (ADA):** Rivier College wants to provide reasonable accommodations to students with disabilities. To accomplish this goal effectively and to ensure the best use of our resources, timely notice of a disability must be provided to the Office of Special Services for verification and for evaluation of available options. Any student whose disabilities fall within ADA should inform the instructor within the first two weeks of the term of any special needs or equipment necessary to accomplish the requirements for the course. To obtain current information on this procedure, contact the Special Services Coordinator at telephone extension 8497.

**Classroom Policies:**
- Active participation requires attendance and arrival to class in time to be prepared for work when the class period begins. Attendance will be taken. If you miss class you are responsible for getting notes from a classmate and notifying the instructor outside of class. Much of the learning will take place in classroom activities that cannot be duplicated easily outside of class. If you miss class, you are responsible for doing all classroom activities you missed, getting the notes from a classmate, and turning in all work on the day it is due. Except in unusual circumstances, students missing more than 20% of the classes (2.8) will fail the course.
- Respect your classmates as well as your instructor. Discussion in class should pertain to the topic of the course and to those asked to speak. All students have a right and responsibility to ask questions and give insight related to the understanding of course content.
- Give and accept constructive criticism. Every student’s solutions will be discussed. We will learn from each others successes and mistakes. At times we will work together to complete a partial solution or give the presenter insight into ways to complete a solution. Presentations of significant attempts at solving a problem will receive some credit.
- E-mail is a great way to keep in touch during the days we don’t meet. You will be expected to acquire e-mail access so that the instructor can notify you of any announcements. You will also be encouraged to dialogue with the instructor and other students via e-mail as a means of improving your mathematical writing and understanding.

**A Few General Problem-Solving Suggestions (many more will be discussed in class!):**
- You should explore each question and write out your thinking in a way that I can follow and you can share with others. Focus on answering the question “why” and communicating your reasons.
- The problems will take time, especially to explore and think. Often you will need to walk away from a problem for a while, and return to the problem for a second or third look before writing it up. Plan on writing rough drafts and ideas first.
- Don’t throw away or erase early attempts. You may discover that they’re helpful in solving the problem.
- Turn in partially worked solutions, even if you say, “I do not understand such and such” or “and then I get stuck here.” Even though you may not have a complete answer to a problem, your written comments will be a valuable learning experience and may lead to a complete solution. Great mathematics often evolves from initial failure.
- Be as specific as possible. Use pictures. Identify all variables. Conjecture.
- Accept at the start that learning necessarily entails some struggling with ideas and feelings of frustration. Stay connected, and see me whenever you are having difficulty or if extenuating circumstances arise.
- The ultimate goal of reasoning is to expand our knowledge, but we cannot seek answers unless we have questions to guide us. We learn, discover, and create by asking questions, so please write down any questions that occur to you as you contemplate the various problems, then bring your written questions to class or e-mail them to me.
Bibliography: In addition to the texts mentioned at the beginning of this syllabus, the following texts have been useful in designing this course and/or may be good resources for you. Sources indicated with an * are on reserve in Regina library.


Problems will be taken from a variety of sources including some of the texts stated above, various websites on problem solving, past Mathematical Olympiads, Putnam exams, mathematics journals, etc. Special recognition should be given to Bruce Cooperstein, UC Santa Cruz, whose Math 30 Introduction to Problem Solving materials were a tremendous help in designing the problem-solving portion of this course. The basketball-based point system, questions for critiquing solutions, and some of the problems came from him.
MA525A Mathematical Problem Solving and Modeling

Major Project: Problem solving in the classroom.

Due no later than 6:30 pm., Tuesday, October 24*.

Problem solving is the heart of mathematics according to mathematician Paul Halmos, but how does one cultivate the art of problem solving in one’s students? Students can be told several strategies (or heuristics) for solving problems, but to become problem solvers they need to experience the process frequently. The National Council of Teachers of Mathematics devotes a section of its Principles and Standards of School Mathematics to problem solving. On page 52, it reads:

Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking…Problem solving is an integrated part of all mathematics learning, and so it should not be an isolated part of the mathematics program.

Recall that a problem does not have an obvious solution (at least to the one attempting to solve it). The solution should require more than a straight forward application of a recently taught formula. The statement of the problem should be open ended enough so that students may consider multiple approaches to solving the problem. At the same time, teachers are faced with the need to cover certain content in each course. Thus problems need to be selected carefully for their ability to motivate and teach mathematical content, tools, and thinking skills without occupying an excessive amount of time.

General assignment: Read pp. 1-7 in Exploratory Problems in Mathematics by Stevenson which is on reserve in Regina Library. Then select an exploratory problem among those on pp. 53-163 of Stevenson’s book than you feel you could use in a specific high school class (exceptions to the source must be approved by the instructor no less than three weeks before the assignment is due). You are encouraged to apply this assignment to a course which you have either observed or taught or are currently observing or teaching. The project write-up need not be a lesson plan (I leave the teaching of lesson plan formats to the education specialists), but you are welcome to include a lesson plan in the project if you’ve already learned how. I am more interested in the selection of a good problem, its appropriateness and effectiveness in a specific secondary mathematics class, and how you would facilitate, assess, and draw on the experience.

Your project must answer the following:

- In what class would you use this activity? Include name of course, grade, level of student, and text.
- At what point of the course would you use this? During/between which topics?
- What content knowledge and mathematical skills would you expect the students to have? Make sure this is consistent with the course you’ve described. Don’t expect them to use knowledge from chapter 12 in chapter 3 (but it may be possible for them to use problem solving as a means of discovering concepts to be discovered later).
- What mathematical concepts do you expect students to learn, recall, or practice through this activity?
- What mathematical learning skills (problem solving, reasoning, proof, communication, connections, representation, modeling, etc.) do you expect students to develop or strengthen through this activity? You should be a little more specific than the suggestions in parentheses.
- Where did you get the problem? Bibliographic reference with page number.
- What is the statement of the problem and how will it be presented to students? If a worksheet will be distributed, include the worksheet. If the problem will be written on the board or stated orally, describe how you will do so. You may change the wording of the problem to make it more appropriate to your class and more captivating to your students. Perhaps you can use the school team or current events to make the problem more interesting. You make choose to add more questions or delete some as you deem appropriate. You might decide to add or delete a picture.
- What are some ways to solve this problem?
- What needs to be done before or as you present the problem? Will the previous lesson lead into the problem, will you give a short introduction, or is it best not to give much of an introduction? Will the students be arranged into groups and of what size? Also describe any manipulatives, materials, or tools beyond pencil and paper that the students will be given or have access to.
- How much time will you allot for this activity? What plans do you have in case you misjudge the timing or some students finish early?
- As the students are working on the problem, what hints (or tangents) will you be prepared to give and under what conditions?
- Will the assessment include individual writings, group reports, observation, and/or class discussion? Consider one of these in more depth by one of the following means:
  - If possible (and with permission), do this activity in a mathematics class. Select three different written solutions (names removed) and describe how you would assess them. Rather than focusing on the correct solution, look for what mathematical and problem solving skills and knowledge the student/group demonstrated. What follow up would be helpful for them?

Read pp. 1-7 in Exploratory Problems in Mathematics by Stevenson which is on reserve in Regina Library. Then select an exploratory problem among those on pp. 53-163 of Stevenson’s book than you feel you could use in a specific high school class (exceptions to the source must be approved by the instructor no less than three weeks before the assignment is due). You are encouraged to apply this assignment to a course which you have either observed or taught or are currently observing or teaching. The project write-up need not be a lesson plan (I leave the teaching of lesson plan formats to the education specialists), but you are welcome to include a lesson plan in the project if you’ve already learned how. I am more interested in the selection of a good problem, its appropriateness and effectiveness in a specific secondary mathematics class, and how you would facilitate, assess, and draw on the experience.
• If possible (and with permission), do this activity in a mathematics class. Record or summarize some of the group discussions that take place. Again discuss what you learn about individual or class knowledge and ability. Did some approaches surprise you? How might you improve the activity?
• Anticipate multiple means of approaching the problem. Develop a rubric to describe what you would expect for A-, B-, and C-level work. This rubric should be tailored to this particular activity but flexible enough to allow multiple approaches.
• What do you expect to observe the students doing? How might your observations affect the next class? What will you do if your expectations are not met? How will you make sure all students meet the objectives?
  o If you haven’t already done so, discuss how later class lessons will draw upon the experience of this activity and whether there will be a follow up to this lesson.

Assessment of this project: See attached rubric. Papers may be refused if one of the following occurs:
1. Due date has passed
2. Requirements have not been met.
3. Grammatical, spelling, or notation errors are noticeable.
4. Paper is not typed, neat, and/or easy to read.

Recommended timeline:
September: Choose problem, course, and chapter. Begin to develop objectives and activity.
Early October: Complete objectives and activity design. Consider possible means of solution. Develop plan for assessment. If incorporating actual student work, use the activity.
Mid- to Late October: Write paper. Proofread. Let someone else read paper and critique. Revise as needed.

Need help? The Rivier College writing center can help with organization and mechanics. The course instructor will help with questions regarding the expectations of this project. You will get more help if you begin the project early and do as much work as possible in advance. The instructor will not answer questions related to selecting a problem/course and getting started less than three weeks before the project is due.

*If you are planning to use this activity in an actual secondary classroom between October 15 and November 19, talk to the instructor in September about adjusting the due dates of the two projects.

Graduate Mathematical Modeling Project

Proposal due October 31
Paper due November 28
Presentations on December 12

Mathematics students should experience mathematical modeling since it helps them relate the mathematics they have learned to real life applications. It can help them value the relevance of mathematics and prepare them to use mathematics beyond the school setting. Some important aspects to have students experience are the simplification of a problem, the identification of important information, the translation of a situation into mathematical language, the use of mathematical techniques, and the final interpretation of the solution. In this course we have focused on discrete dynamical systems, but one can also mathematical functions and discrete graphs as models.

Step I: Identify a project.
Option 1: Many situations can be modeled mathematically. In class, we have experimented with situations where a fixed quantity or percentage is added or subtracted at each stage to obtain the new quantity. In chapter two, we will look at how randomness affects our models, and in chapter three, we will look at multi-state problems where objects can move from one state to another (an example of this is “child”, “teen”, “young adult”, “midlife”, and “senior” with respect to population over time). Think of a situation that might be modeled mathematically in one or more of these ways. Recall that modeling is representation with a purpose (beyond completing a math project). Think about what questions your mathematical model would address. You may want to think about a compartmental diagram. Identify
which influencing factors you want to consider. You are encouraged to select a model which would be of interest to secondary school students or to choose a situation relating to mathematics education.

Option 2: Create an activity that you could use to help secondary school students develop modeling skills. Will you provide the data sets or have students collect data? How will the students decide what information is pertinent? What knowledge of mathematical models will they have prior to the activity? What are the objectives? How will they be assessed? You may adapt an activity that you find (NCTM’s Math Teacher is one possible source) or expand a problem/project in a text, but make sure you make this project your own in a significant way. Refer to the requirements of the problem-solving assignment for expectations if you select this option. Note the difference between problem solving and modeling as you develop the two projects.

**Step IIA: Discuss your idea(s) with the instructor.** She will help to make sure your model or lesson fits the objectives of this assignment and may be able to suggest strategies and contacts.

**Step IIb: Find information.** Collect the data you need. For example, if you wanted to study the fulltime student population at Rivier College, contact appropriate administrators to determine the average number of admitted first year students over the last few years, the average number who graduate annually, the percentage who leave or switch to part time, the typical number of transfer students, etc. If you’re interested in environmental issues, you might contact someone at a water protection/monitoring agency to find out what kind of pollutants are a concern at a particular water source, what affects the level of pollutant, flow in/flow out, etc. Websites may be of help, but it may be a challenge to find the right one. The US Census website does have a lot of statistics that may be of help in some projects. Robin Lock of St. Lawrence University has a nice website [http://it.stlawu.edu/%7Erlock/](http://it.stlawu.edu/%7Erlock/) with links to databases. If you’ve chosen option 2, look in NCTM journals, library books with mathematics projects, secondary textbooks, etc.

**Step III: Develop the Model or Lesson.**

Option 1: Use the data over a few time intervals to develop a model for the situation. Remember that a model should be simpler than reality, but accurate enough to be useful in the short run. What assumptions have you decided to make in developing your model? Perhaps you might want to focus on one factor first and then consider additional factors. Or you might want to consider best case and worst case scenarios. Maybe you could consider “what ifs”—possible remedies to an existing problem such as we did with the bobcat population.

Option 2: How do you help students develop the model? What hints/formulae/tools are provided? How does the activity relate to recent lessons? Prepare any worksheets or assignments you would give. List objectives for the activity. Determine the class for which the activity would be designed. Is there a particular point in the course where it fits best? What are the students’ backgrounds? How much time will you allot?

**Step IV: Address the problem.**

Option 1: What problem does your model help you address? What answers does it provide?

Option 2: Be sure to ask the students to begin with a real life problem, determine how to solve it mathematically, and then write up solutions in nonmathematical terms. What other work would you want to receive from the students? If the activity is done with an entire class, how do you involve and assess students?

**Step V: The Paper.**

Option 1: Write a paper describing what you did. You may address the paper to a general audience or to a fictitious (or real?) client who may have posed the question. Describe the problem, the data, the assumptions, your development of the model, the model itself, and the extent to which it addressed the problem. Try to use words that a non-mathematician would understand. Attach any computer trials or computations. Include data summaries and computations in a briefly, orderly, and informative manner. (I don’t need to see your scratch work. See the top of p. 67 for a nice use of computations in text.)

Option 2: Write a paper describing what you did. You will probably want to include a lesson plan. See Problem Solving assignment for additional details on lesson plan expectations in this class.

**Step VI: The Presentation.** Give an oral presentation (15-20 minutes) on December 12 to the class.
Option 1: Introduce the problem, the data collection, your assumptions, your development of the model, the model itself, and how the model addressed the problem. Use visuals such as a compartmental diagram, graph of data, graph of model, and cobweb plot to help illustrate your work. Not all of these visuals will be appropriate for some models. You don’t need to demonstrate “routine” computations although you may be asked to do some to verify points you made. While maintaining a professional manner, address the mathematical level of your talk to the audience who will be there.

Option 2: Describe the class, objectives, activity, and assessment. Identify the source of the project and to what extent you personalized it. Why did you choose this activity and how does it meet the objectives of mathematical modeling and the needs of the secondary math class? Do not ask our class to do the activity, but if you tried your activity out in a classroom you may summarize the results.

Option 3: Give a presentation on your problem-solving project. See preceding paragraph.

Assessment: Both the paper and the presentation will be assessed for mathematical correctness, level of work in developing the model or activity, organization, clarity, and appropriate use of visuals. In addition, the paper will be assessed for spelling, grammar, and use of notation. The presentation will also take into account style of delivery: Speak up, face the class, avoid slang, write clearly, and speak with confidence. In both, a clear introduction is essential.

525A Rubric for Problem Solving Project

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<tr>
<th>Selection of Problem</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>F</th>
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<tbody>
<tr>
<td>Problem is open-ended enough to allow multiple approaches yet fits naturally with mathematical content of course. Activity promotes development of problem-solving skills and math content objectives. Expectations are appropriate for course and level.</td>
<td>The problem chosen meets most but not all of the criteria described for an A. It promotes either the problem-solving or content objectives well, but is not quite as strong in some areas. A good problem may need some revision to make it work for the described class.</td>
<td>Problem meets many, but not all, of the criteria in A or the appropriateness is mediocre. Problem chosen may be too easy, too hard, uninspired, or too weak to promote mathematical and problem-solving objectives.</td>
<td>Problem does not encourage students to use or develop problem-solving skills. Connection between problem and course content may be weak.</td>
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<tr>
<th>Presentation of Activity</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>F</th>
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<tr>
<td>Writer has clarified how the activity will proceed. Connections to course content are made evident by the writer. Worksheet or oral instruction is appealing, clear, and understandable to the student. Selection of group structure and tools is appropriate for problem. Contingency plans and hints have been considered.</td>
<td>The writer presents has nice ideas for presenting the activity, but some of the plans have not been developed fully or may not be the best for the chosen course or problem.</td>
<td>Ideas for activity are okay, but not adequately delineated. Your discussion of some of the items related to the activity is either missing or too vague. You need to think more deeply about how the activity might proceed in a secondary classroom and the questions students will have.</td>
<td>You may have selected a problem and course, but it is not clear how the activity would proceed. Wording of problem may be poor and hard for students to understand. You have not considered what might actually occur in a classroom.</td>
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<th>Assessment</th>
<th>A</th>
<th>B</th>
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<tr>
<td>Writer has fully developed an assessment appropriate to activity and objectives. Multiple approaches to solving the problem have been considered. Assessment can be used for improvement of lesson, amending future lessons, or individual improvement of students.</td>
<td>Assessment plans are appropriate to activity and objectives, but either the paper do not clearly convey how the assessment will be conducted and used or the assessment does not allow for some problem solving approaches.</td>
<td>Plans for assessment are present, but are minimally developed. Some objectives or alternative approaches may have been overlooked.</td>
<td>Plans for assessment are missing, too vague, or not directly connected to project and objectives. Alternative approaches to solving the problem have not been considered.</td>
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<th>Originality and Depth</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>F</th>
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<tr>
<td>Writer has gone beyond the basic assignment by one or more of the following: revision of problem, creative</td>
<td>The paper addresses the requirements, but does not go substantially beyond them. Perhaps the problem</td>
<td>A few requirements have not been fully addressed or discussions are minimally adequate. Writer might</td>
<td>Discussion is weak. Writer has not met many requirements fully. Writer tends to rely on material</td>
<td></td>
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<tr>
<td>Organization</td>
<td>Information in paper is presented in a clear and logical manner. All project criteria are easy to find while maintaining a nice flow in the paper. Paragraphs are coherent and appropriately used.</td>
<td>There may be a few minor errors in paragraph structure and arrangement of content, but overall the paper is well written. Perhaps some references are unclear or some of the writing is choppy.</td>
<td>Presentation needs more coherence. There may be some errors in paragraph structure. Your style of writing might not generate interest in the topic.</td>
<td>Project is messy and disorganized. Connections between sections are unclear. Discussions lack coherent paragraph structure.</td>
</tr>
<tr>
<td>Mechanics</td>
<td>Bullets and headings are consistent throughout. Spelling, grammar, and mathematical notation are correct.</td>
<td>Very few errors in spelling, grammar, bullets, or notation. Errors are minor and do not interfere with clarity of paper.</td>
<td>Some, but not many, minor errors and very few major errors in mechanics. One or two may affect clarity.</td>
<td>Repeated minor errors or several major errors in spelling, grammar, or notation interfere with reading of paper. Sloppy paper. Inconsistent style.</td>
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