A GREAT discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest, but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.

--George Pólya from How to Solve It (Princeton 1945).

Instructor: Dr. Terri Magnus, Regis Hall (first floor)
Contact information: 897-8462, tmagnus@rivier.edu
Office hours: Please let me know whenever you need help. The further you get behind, the harder it is to catch up. You are invited to stop by my office whenever I am there or make an appointment. I will also respond to e-mail questions. My scheduled office hours are T10am-12pm, 5-6 pm, W 9:30-11:30am, R4-6pm. I will also be available just before and after class.

Textbook:

✓ For the modeling portion of the course, the text A Course in Mathematical Modeling by Douglas Mooney and Randall Swift, The Mathematical Association of America, 1999, is available from the Rivier College bookstore. You may find it useful to have a Calculus text and a spreadsheet program such as Excel available as well.
✓ For the problem-solving portion of the course, no text is required. It is recommended that students have access to a Discrete Mathematics text such as Rosen’s Discrete Mathematics and its Applications (5th edition) Boston: WCB/McGraw-Hill, 2003. Students not owning a Discrete Mathematics text or wishing to have a text more closely aligned with this course may obtain The Art and Craft of Problem Solving by Paul Zeitz, NY:John Wiley and Sons, Inc., 1999. These texts (5th ed. of Rosen), along with other problem-solving texts, will be available through the library reserve desk.

Course description from catalogue: The purpose of this course is two-fold. The first is to study various mathematical problem solving strategies, including such topics as symmetry, extrema, pigeonhole principle, invariants, complex variables, algebraic manipulation, and generating functions. The second aim is to develop the techniques of discrete dynamical modeling. Applications to illustrate the techniques are drawn principally from the natural and social sciences. Each student creates, analyzes, refines, and presents a mathematical model from a field of his or her own choosing. Prerequisites: MA161/166 and MA310.

Instructor’s elaboration on course description: The problem-solving thread of the course will be devoted to learning a variety of problem-solving heuristics and mathematical problem-solving techniques as students work on problems both individually and in small groups. Students will practice reading problems carefully, determining an appropriate technique for attacking each problem, and writing up their solutions.

The second focus of the course will be on mathematical models. After a brief introduction, students will study discrete dynamical systems and determine fixed points analytically and graphically. Fitting curves to data will also be discussed. If time permits, the application of differential equations to mathematical modeling will be explored.

Course Objectives:
A student completing this course should be able to...
• read a problem and determine a reasonable approach to solving the problem.
• understand and employ a variety of problem-solving heuristics.
• apply techniques and knowledge from number theory, graph theory, probability, geometry, and other mathematical fields to problem solving.
• write clear and complete solutions to problems.
• demonstrate problem-solving skills to others
• develop discrete and continuous mathematical models and use them to understand a phenomenon outside of mathematics.

Teaching Strategies:
• Students actively solving problems individually and in small groups.
• Problem exploration and solving both in class and as homework.
• Types of problems intermixed so that students develop the ability to choose appropriate means to solving them.
• Student write-ups of problem solutions.
• Reflections on problem solving process.
• Student presentation and constructive critique of problem solutions.
• Lectures only as needed to clarify problem solving techniques and heuristics, and to introduce modeling.
• Students completing and presenting a mathematical modeling project.

Course Requirements:

• Attendance, active participation, and engagement in full-class, small-group, and individual activities.
• Full write-up of a suggested number of mixed-type problem solutions.
• Oral presentation of some solutions to the class.
• Group or individual modeling project.
• Homework problems based on material and techniques covered in class.
• Oral final examination.

Final grades: Students will accumulate points through the various assignments.

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixed problems</td>
<td>35</td>
</tr>
<tr>
<td>Focused homework problems</td>
<td>20</td>
</tr>
<tr>
<td>Oral presentations/participation</td>
<td>10</td>
</tr>
<tr>
<td>Modeling project</td>
<td>25</td>
</tr>
<tr>
<td>Oral final exam</td>
<td>10</td>
</tr>
</tbody>
</table>

A (93-100 points), A- (92-90 points), B+ (89-87 points), B (83-86 points), B- (80-82 points), C+ (77-79 points), C (73-76 points), C- (70-72 points), D+ (67-69 points), D (63-66 points).

Mixed Problems: Frequently, you will be presented with a list of mixed problems. The problems will be taken from a variety of sources and require a variety of techniques to be solved. These problems will often not be based only on the techniques just discussed in class. You are not expected to solve all of the problems, but should make your best effort to complete and write up some of the solutions each week. Submitted solutions must be legible and include a statement of the problem, the number of the list that contained the problem, and the number of the problem itself. Normally a word or numerical value will not suffice for a solution; you must use full sentences as you demonstrate (prove) why your solution is correct. Problems have been assigned a point value (1 for free throw, 2 for field goal, 3 for beyond the arc) based on the anticipated level of difficulty. You may go back and solve problems from earlier lists, provided the solution has not already been presented to the class. You are also allowed to give alternative solutions to previously presented problems provided that your solution differs substantially from the solution presented earlier. The instructor retains the right to determine when partial solutions deserve credit and when outstanding solutions deserve additional credit. The instructor will give feedback on incomplete solutions and allow resubmissions until someone in the class has solved the problem. Aim to earn approximately 2-3 points per week over the semester. Of course, you are welcome to do more sooner!

Homework Problems: After techniques in mathematical problem-solving and modeling have been discussed in class, the instructor may give an assignment related to the material. These problems will be due the following class day. You are encouraged to contact the instructor prior to the due date when you have questions on the assignment. Once again solutions must be legible, complete, and written in full sentences. Good proof writing style is expected. The statement (or your restatement) of the problem should precede the solution.

Oral presentations of problems: Students will be expected to present some solutions and partial solutions to the class. Classmates will then discuss the presentation considering some of the following questions: Is the solution persuasive? Has the solution been communicated effectively? Is the argument sufficiently thorough? Have all factors been taken into account? Can a more general problem be solved? Are there alternative approaches? Can the proof be simplified? Students may earn additional points by significantly contributing to the discussion. Normally, oral problem presentations will be worth 1-3 points each depending on the difficulty, progress, and thoroughness. Students may also earn points for presenting partial solutions and significant attempts at solving problems. Students should contact the instructor (via e-mail) prior to class indicating their desire to present, their problem of choice, and the extent to which they have solved the problem.

Modeling project: In early December, each student will complete and present a project in which they choose a situation outside of mathematics (environmental studies, economics, traffic flow, social demographics, etc.) and develop and analyze it using mathematical modeling techniques. Details of the project requirement will be handed out to students at a later date, but students will be expected to present their project both orally and in writing.

Final exam: Students will schedule a 30-minute individual appointment in early December during which they will discuss their understanding of mathematical problem solving and modeling. The instructor will give suggestions at least a week in advance regarding topics the students should review. Students may participate in the national Putnam problem-solving competition on Saturday, December 1, in lieu of the oral final exam.
**Extra points:** It is possible for students to exceed the specified number of points in the categories of mixed problems and oral presentations. These points may be used to offset lower totals in other categories provided that the student earns at least half of the points within that category. In particular, every student is expected to give some oral presentations, to complete the modeling project, to do homework assignments, and take the oral exam, and must earn at least 50% in each of these categories before points beyond 35 from the mixed problems category would be allowed to improve the average.

**Running Total?** Feel free to e-mail or visit the instructor to verify your current total and distribution of points at any time during the semester. The instructor will keep a chart indicating the problems you have earned points on. You are encouraged to do the same.

**Americans with Disabilities Act (ADA):** Rivier College wants to provide reasonable accommodations to students with disabilities. To accomplish this goal effectively and to ensure the best use of our resources, timely notice of a disability must be provided to the Office of Special Services for verification and for evaluation of available options. Any student whose disabilities fall within ADA should inform the instructor within the first two weeks of the term of any special needs or equipment necessary to accomplish the requirements for the course. To obtain current information on this procedure, contact the Special Services Coordinator at telephone extension 8497.

**Classroom Policies:**
- Active participation requires attendance and arrival to class in time to be prepared for work when the class period begins. Attendance will be taken. If you miss class you are responsible for getting notes from a classmate and notifying the instructor outside of class. Much of the learning will take place in classroom activities that cannot be duplicated easily outside of class. If you miss class, you are responsible for doing all classroom activities you missed, getting the notes from a classmate, and turning in all work on the day it is due. Except in unusual circumstances, students missing more than 20% of the classes (2.8) will fail the course.
- Respect your classmates as well as your instructor. Discussion in class should pertain to the topic of the course and to those asked to speak. All students have a right and responsibility to ask questions and give insight related to the understanding of course content.
- Give and accept constructive criticism. Every student’s solutions will be discussed. We will learn from each others successes and mistakes. At times we will work together to complete a partial solution or give the presenter insight into ways to complete a solution. Presentations of significant attempts at solving a problem will receive some credit.
- E-mail is a great way to keep in touch during the days we don’t meet. You will be expected to acquire e-mail access so that the instructor can notify you of any announcements. You will also be encouraged to dialogue with the instructor and other students via e-mail as a means of improving your mathematical writing and understanding.

**A Few General Problem-Solving Suggestions (many more will be discussed in class!):**
- You should explore each question and write out your thinking in a way that I can follow and you can share with others. Focus on answering the question “why” and communicating your reasons.
- The problems will take time, especially to explore and think. Often you will need to walk away from a problem for a while, and return to the problem for a second or third look before writing it up. Plan on writing rough drafts and ideas first.
- Don’t throw away or erase early attempts. You may discover that they’re helpful in solving the problem.
- Turn in partially worked solutions, even if you say, “I do not understand such and such” or “and then I get stuck here.” Even though you may not have a complete answer to a problem, your written comments will be a valuable learning experience and may lead to a complete solution. Great mathematics often evolves from initial failure.
- Be as specific as possible. Use pictures. Identify all variables. Conjecture.
- Accept at the start that learning necessarily entails some struggling with ideas and feelings of frustration. Stay connected, and see me whenever you are having difficulty or if extenuating circumstances arise.
- The ultimate goal of reasoning is to expand our knowledge, but we cannot seek answers unless we have questions to guide us. We learn, discover, and create by asking questions, so please write down any questions that occur to you as you contemplate the various problems, then bring your written questions to class or e-mail them to me.

**Bibliography:** In addition to the texts mentioned at the beginning of this syllabus, the following texts have been useful in designing this course and/or may be good resources for you. Sources indicated with a * are on reserve in Regina library.


Problems will be taken from a variety of sources including some of the texts stated above, various websites on problem solving, past Mathematical Olympiads, Putnam exams, mathematics journals, etc. Special recognition should be given to Bruce Cooperstein, UC Santa Cruz, whose Math 30 Introduction to Problem Solving materials were a tremendous help in designing the problem-solving portion of this course. The basketball-based point system, questions for critiquing solutions, and some of the problems came from him.